Local Temperatures in Semigray Nondiffuse Cones and V-grooves

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Theme

THE temperature distributions for cavities with an apex are investigated analytically. Radiant exchange is assumed to be the only effective mode of heat transfer. Higher-order interreflections of radiant energy will play an important role in determining the temperature near the apex. Therefore, in order to evaluate the extreme temperatures at the apex, accurate solutions are required of semigray radiosity integral equations for this configuration.

Contents

The present analysis is mainly carried out for circular conical cavities. Consider a ray from P_i to P_j via one specular reflection at P_I, as shown in Fig. 1. The point O is the apex, and φ is the half angle of the cone. A point $P_j{}'$ is the image of P_j concerning the plane mirror which is tangent with the cone at P_I. It holds that the angles $\angle P_iOP_I$ and $\angle P_IOP_j$ are equal, and the line OP_I is the bisector of the angle $\angle P_iOP_j{}'$. Similarly, consider $n-1$ plane mirrors for $n-1$ specular reflections. Each of the mirrors is tangent with the cone at each of the points of intermediate contact, $P_1, P_2, \ldots, P_{n-1}$. As shown in Fig. 1, we can see along the line P_iP_I the point P_I directly, the image $P_2{}'$ of P_2 through the mirror at P_I and the $(n-1)$th image $P_j^{(n-1)}$ of P_j through $n-1$ mirrors. The angles $\angle P_iOP_1$, $\angle P_1OP_2, \ldots,$ $\angle P_{n-1}OP_j$ are equal to a common value t, and the line OP_I is one of n-sectors of the triangle $P_iOP_j^{(n-1)}$. Let the angular displacement of P_j from P_i, and that of P_I be θ_j and θ_I. The displacement θ_j is shown to be n times θ_I, and the value of t is related with θ_I as

$$1-(1-\cos\theta_I)\sin^2\varphi=\cos t \tag{1}$$

It should be noticed that $\cos t$ of Eq. (1) is equal to $K/2$ of Ref. 1. The relation between the radial coordinates r_i, r_I, and r_j of P_i, P_I and P_j, respectively, are as follows,

$$(\sin t/r_j)=(\sin nt/r_I)-[\sin(n-1)t]/r_i] \tag{2}$$

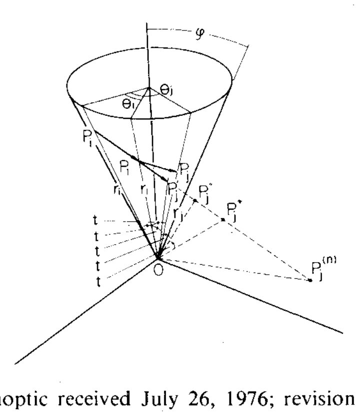

Fig. 1 Specular reflection in a conical cavity. Images $P_j{}'$, $P_j{}''$, ..., $P_j^{(n-1)}$ are seen along line of sight $P_i - P_I$. $< P_iOP_j^{(n-1)} = nt$.

Equation (2) is essentially the same as that reported in Ref. 1, but gives a far simpler expression for the exchange factor suitable to the kernel of the radiosity equation. Using Eq. (2) and following the method of Ref. 1, we can obtain the required exchange factor. In the following, the incident solar flux is supposed to be parallel with the cone axis. Let dA and dA' be the ring area elements at r and r', respectively. The exchange factor $dE_{dA-dA'}$ between dA and dA' is expressed as

$$dE_{dA-dA'}=\frac{2}{\pi}\cdot\frac{\cos^2\varphi}{\sin\varphi}r\,r'^2dr' \times\sum_{n=1}^{\infty}(\rho_s)^{n-1}\int_{t=0}^{t_{\max}}\left(\frac{\sin nt}{\sin t}\right)^3 \cdot\frac{(1-\cos t)^{3/2}\sin t\,dt}{(r^2+r'^2-2r\,r'\cos nt)^2\cdot(2\sin^2\varphi-1+\cos t)^{1/2}} \tag{3}$$

where ρ_s is the specular component of reflectance. Because the value of t must be within $[0,2\varphi]$ and $[0,\pi/n]$, $t_{\max}$ is equal to $\min(2\varphi,\ \pi/n)$. The derivative of the previous exchange factor with respect to r' is the kernel, $K(r,r')$, of the radiosity equation. The kernel $K(r,r')$ behaves generally as Dirac's delta function, $\delta(r')$, at the intersection of plates,

$$\lim_{r\to 0}K(r,r')=M\cdot\delta(r') \tag{4}$$

where $M=\lim_{r\to 0}E_{dA-A}=E_{O-A}$ and E_{dA-A} is the exchange factor between the area element dA at r and the total inside surface A of the cone. By virtue of Eq. (4), the solution of the semigray radiosity equation[2] at the intersection is exactly obtained, and the value of the equilibrium temperature is given by

$$\sigma T(0)^4=\frac{\alpha^*S\cdot E_s{}^*(0)\,(1-\rho_d\cdot E_{O-A})}{\epsilon(1-\rho_d{}^*\cdot E_{O-A}{}^*)\cdot[1-(\rho_d+\epsilon)\cdot E_{O-A}]} \tag{5}$$

where σ, S, α, ϵ, and ρ_d are the Stefan-Boltzmann constant, solar constant ($1.395 KW/m^2$) absorptivity, emissivity, and diffuse reflectance, respectively. The quantities associated with solar radiation are distinguished from those of surface thermal radiation by an asterisk*. The quantity $E_s^*(0)$ is the exchange factor associated with the external solar flux. For circular conical cavities with a diffusely reflecting surface, M is found to be $1-\sin^3\varphi$, and $E_s^*(0)$ is equal to $\sin\varphi$. Therefore, in this case, the temperature is given by

$$\sigma T(0)^4=\frac{\alpha^*S[1-\rho_d(1-\sin^2\varphi)]}{\epsilon[1-\rho_d{}^*(1-\sin^3\varphi)]\cdot\sin^2\varphi} \tag{6}$$

For V-groove cavities of half angle φ, we have $M=\cos^2\varphi$, and

$$\sigma T(0)^4=\frac{\alpha^*S(1-\rho_d\cos^2\varphi)}{\epsilon(1-\rho_d{}^*\cos^2\varphi)\sin\varphi} \tag{7}$$

The expressions of the values at the apex are reported in Ref. 3 for V-groove cavities. Equation (7) is also obtained

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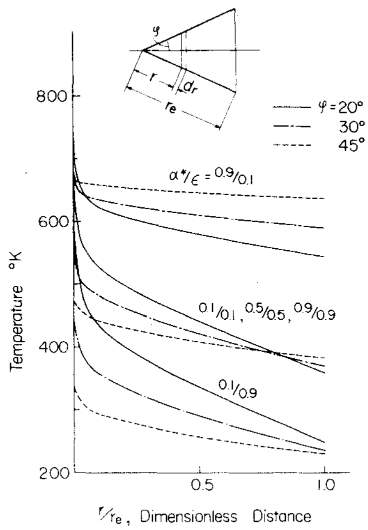

Fig. 2 Temperature distribution for a circular conical cavity with a diffuse reflecting surface.

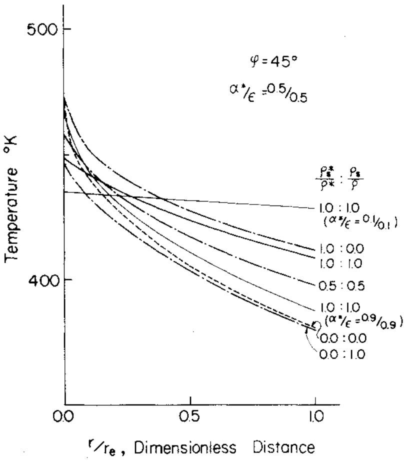

Fig. 3 Effect of specular reflection on the temperature distribution for circular conical cavities.

with these expressions. By use of the estimated value at the apex, the numerical solution of the semigray radiosity equation is more easily obtained over the cone. In Fig. 2, the equilibrium temperature distribution is shown for conical cavities with a diffusely reflecting surface. The r is the distance measured along the surface from the apex and φ is the cone-half angle. The temperature, $T(r)$, increases sharply as r approaches 0, when φ is small and the ratio, α^*/ϵ, is small, while it is rather flat for large φ and all α^*/ϵ. A fixed value of α^*/ϵ gives the same temperature distribution, which is also seen in the V-groove cavities.[2] In Fig. 3, the effect of the specular component of reflectance on the temperature distribution is shown for $\alpha^*/\epsilon = 1$, and $\varphi = 45°$. The value of $T(r)$ at the open end is influenced more by ρ_s^* than by ρ_s and is the largest for the surface specular to the solar radiation. At the apex, on the other hand, $T(r)$ is influenced more by ρ_s and the largest for a surface diffuse to the thermal radiation. Except for the case of $\alpha^*/\epsilon = 0.9/0.9$, the most flat distribution is found for a specularly reflecting surface, $\rho_s^* = \rho_s = 1.0$. The temperature distributions are affected by the specular component of reflection for a fixed value of α^*/ϵ. When $\varphi < \pi/4$, though not presented here, the discontinuities in temperature distribution appear due to those of $E_s^*(r)$ similarly to V-groove cavities.

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